Pearson Edexcel Level 3

GCE Further Mathematics

Advanced Subsidiary

Further Mechanics 1

Specimen paper **Time: 50 minutes**	**Paper Reference(s)** **8FM0/25**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 4 questions in this section of the paper. The total mark is 40.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Unless otherwise indicated, whenever a value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

Answer ALL questions. Write your answers in the spaces provided.

1. A ball is projected with speed 6 m s^{-1} up a line of greatest slope of an inclined plane. The plane is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{7}$. The ball is modelled as a particle, the plane is modelled as being smooth and air resistance is modelled as being negligible.

Using the conservation of energy principle, find the speed of the ball at the instant when it has travelled a distance of 5 m up the plane.

(Total for Question 1 is 5 marks)

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2. A particle P of mass m is moving in a straight line on a rough horizontal table. The particle collides with a fixed vertical wall. Immediately before P collides with the wall, P is moving with speed u in a direction that is perpendicular to the wall. In the collision, P receives an impulse of magnitude $\frac{5mu}{3}$.

After the collision, the total resistance to the motion of P is modelled as a constant force of magnitude $\frac{mg}{6}$.

(a) Find, in terms of u and g, the distance of P from the wall when P comes to rest.

(7)

(b) State how the model for the total resistance could be refined to make it more realistic.

(1)

(Total for Question 2 is 8 marks)

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3. A car of mass 500 kg is moving at a speed of v m s^{-1}. The total non-gravitational resistance to the motion of the car is modelled as having magnitude $(5v + C)$ newtons, where C is a constant.

The car moves up a straight road which is inclined to the horizontal at an angle θ, where $\sin\theta = \frac{1}{14}$. When the engine of the car is working at a constant rate of 25 kW, the car is moving up the road at a constant speed of 20 m s^{-1}.

(a) Find the value of C.

(5)

With the engine of the car again working at 25 kW, the car now moves along a straight horizontal road at a constant speed of U m s^{-1}.

(b) Find the value of U, giving your answer to 2 significant figures.

(6)

(Total for Question 3 is 11 marks)

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4. A particle P of mass m is at rest on smooth horizontal ground between two fixed parallel vertical walls. Another particle Q of mass m is moving in a straight line along the ground between the walls in a direction which is perpendicular to the walls. Particle Q collides with particle P directly. The coefficient of restitution between the particles is e and the speed of Q immediately before the collision is u.

(a) Show that the speed of P immediately after the collision is $\frac{u}{2}(1 + e)$.

(6)

(b) Find the speed of Q immediately after the collision.

(1)

Given that the total kinetic energy lost in the collision between the two particles is $\frac{3mu^2}{16}$,

(c) find the value of e.

(5)

Suppose now that the coefficient of restitution between the particles is 1 and that the coefficient of restitution between each particle and each wall is 1.

(d) By considering at least two collisions between the particles, describe in detail what happens in the subsequent motion, giving reasons for your answer.

(4)

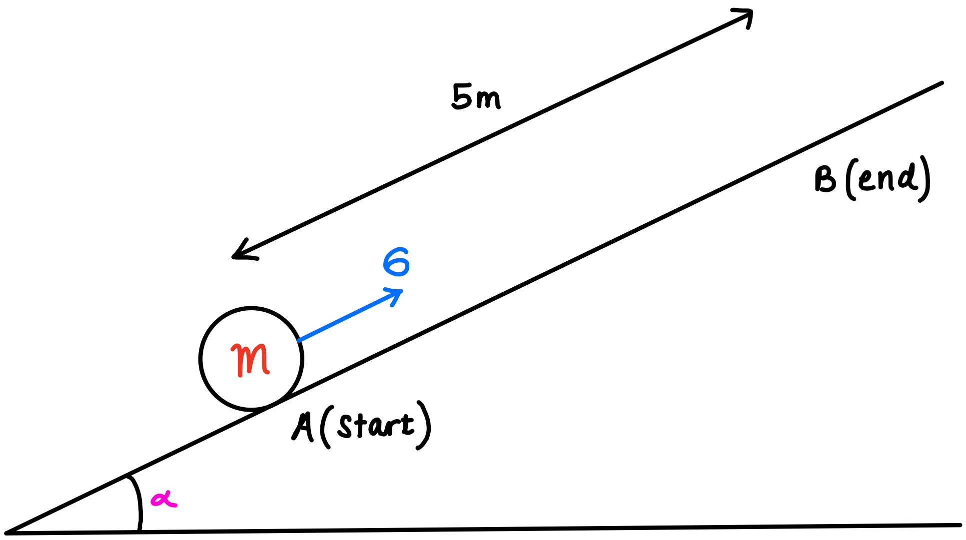
(Total for Question 4 is 16 marks)

TOTAL FOR PAPER IS 40 MARKS

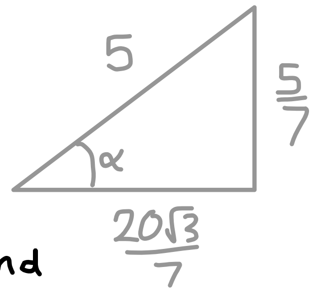
1) Set up a diagram labelling all relevant forces.

$\sin\alpha = \frac{1}{7}$

$g = 9.8\,ms^{-2}$



We can use trigonometry to find the change in vertical height using the value of $\sin\alpha$ and the distance moved up the ramp:



Now using a table, we can calculate the energy before and after using the law of conservation of energy.

	Potential Energy	Kinetic Energy
Start	0	$\frac{1}{2}m(6)^2$
End	$\frac{5}{7}mg$	$\frac{1}{2}m(v)^2$

Using conservation of energy we can form the equation that:

Energy before/start = Energy after/end

$$\therefore \frac{1}{2}m \times 6^2 = \frac{5}{7}mg + \frac{1}{2}mv^2$$

We can now simplify and rearrange the equation to solve for v.

$$18m = \frac{5}{7}mg + \frac{1}{2}mv^2$$

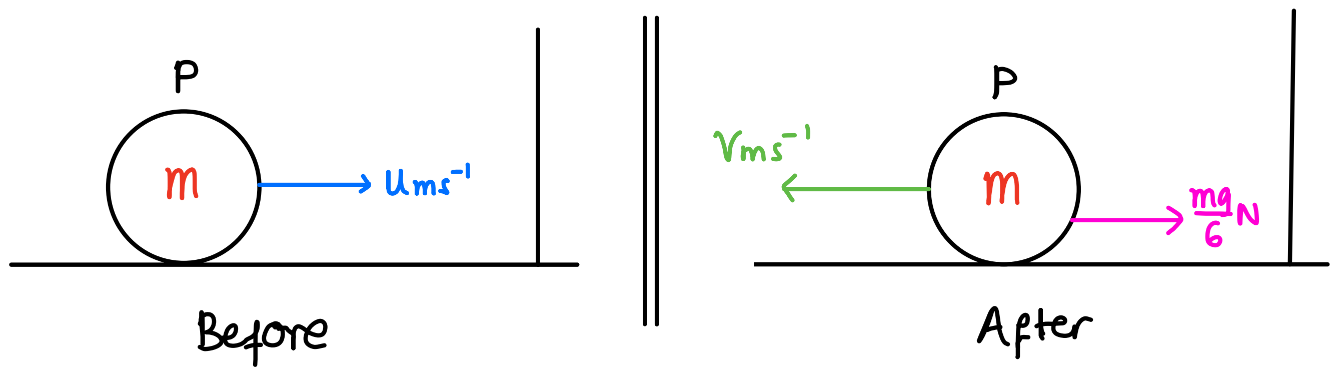
$$18 = \frac{5}{7}g + \frac{1}{2}v^2$$

$$\frac{1}{2}v^2 = 18 - \frac{5}{7}g$$

$$v^2 = 36 - \frac{10}{7}g$$

$$v = \sqrt{36 - \frac{10}{7}(9.8)} = \sqrt{22} \approx 4.69\,ms^{-1}\ (3.s.f)$$

2) a) We can draw a diagram of each balls' velocity before and after as such:



To calculate impulse, we take away the initial momentum from the final momentum which essentially is the change in momentum where momentum is mass multiplied by velocity.

$\therefore$ $\underline{I} = m\underline{v} - m\underline{u}$ so we substitute the values in and solve for v.

$$|\underline{I}| = \frac{5mu}{3} = mv - m(-u)$$

$$\frac{5u}{3} = v + u$$

$$v = \frac{5u}{3} - u = \frac{5u}{3} - \frac{3u}{3} = \frac{2u}{3} ms^{-1}$$

$\therefore$ Since work done by friction is equal to kinetic energy after collision.

$\therefore$ $F \times d = \frac{1}{2}mv^2$ and now we substitute and rearrange for d.

$$\frac{mg}{6} \times \text{distance travelled} = \frac{1}{2}mv^2$$

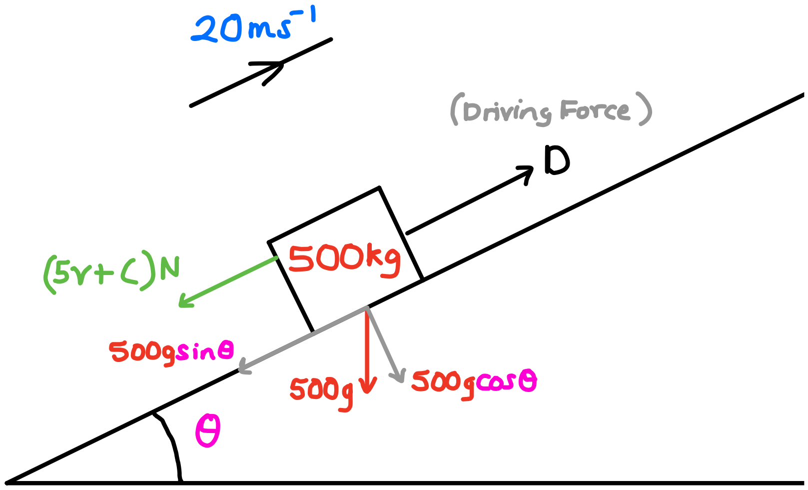
$$\frac{mg}{6} \times d = \frac{1}{2}m\left(\frac{2u}{3}\right)^2$$

$$\frac{g}{6} \times d = \frac{1}{2} \times \frac{4u^2}{9}$$

$$\frac{g}{6} \times d = \frac{2u^2}{9} \Rightarrow d = \frac{2u^2}{9} \times \frac{6}{g} = \frac{12u^2}{9g} = \frac{4u^2}{3g}$$ //

b) The total resistance could be depend on velocity or have some sort of variable friction.

3) Set up a diagram labelling all relevant forces.



$\sin\theta = \frac{1}{14}$

Power = 25kW
= 25000W

a) To calculate the driving force we use the equation : $P = Fv$, where the force (F) is the driving force (D)

$P = Fv$, where $F = D$

$\therefore D = \frac{P}{v} = \frac{25000}{20} = 1250\,N$

Since speed is constant, the sum of the forces is equal to zero since when we resolve forces parallel to the plane where $\Sigma F = ma$. However since the velocity is constant, acceleration is zero $\therefore \Sigma F = 0$ $\therefore$ We can say :

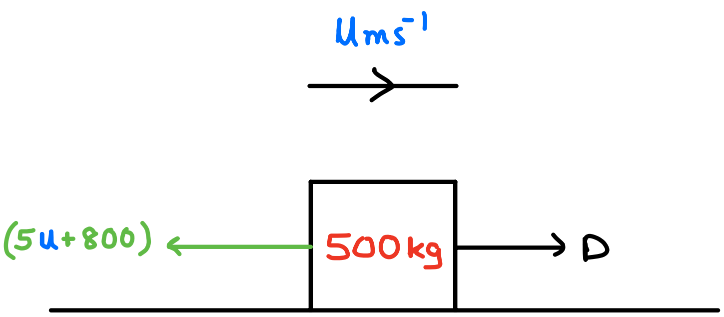
$\therefore 1250 - 500g\sin\theta - (5v + C) = 0$

$1250 - 500(9.8)(\frac{1}{14}) - (5(20) + C) = 0$

$1250 - 350 - 100 - C = 0$

$\therefore C = 1250 - 350 - 100 = 800N$ //

b) We should draw a new diagram now considering the change in incline and the new resistive force so we use the equation $P = Fv$, where the force (F) is the driving force (D).



$\therefore P = Dv$

$\therefore D = \frac{P}{v} = \frac{25000}{u}$

Since velocity is constant, the sum of the horizontal forces is zero.

$$\therefore \frac{25000}{u} - (5u + 800) = 0$$

$$\frac{25000}{u} - 5u - 800 = 0 \quad \Big) \times u$$

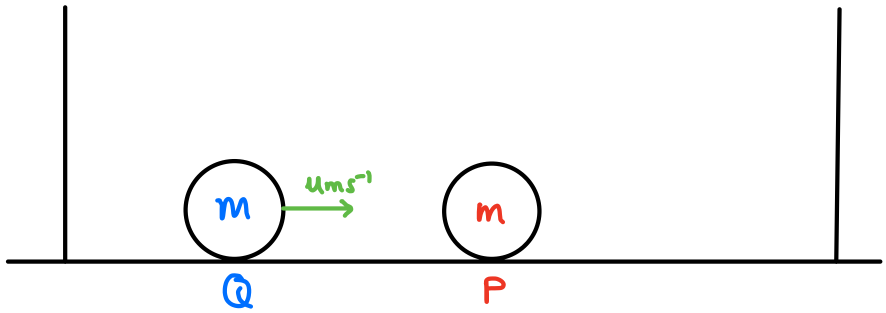
$$25000 - 5u^2 - 800u = 0$$

$$5u^2 + 800u - 25000 = 0$$

$\therefore u = 26.77078252$ ✓ $\quad \therefore u = 27\,ms^{-1}$ (3.s.f)

$u = -186.7707825$ × $\quad$ as u must be positive $(u > 0)$.

4) a)



Using conservation of linear momentum we can say:

$$m(u) + m(0) = mV_Q + mV_P$$

$u = V_Q + V_P \quad V_Q = u - V_P$, where V_P is the velocity of P after the collision and the same for V_Q.

Using Newtons experimental law, we can find V_P.

$$e = \frac{\text{speed of seperation}}{\text{speed of approach}} = \frac{V_P - V_Q}{u}$$

$$\therefore eu = V_P - V_Q = V_P - (u - V_P)$$

$$eu = -u + 2V_P$$

$$2V_P = u + eu$$

$$V_P = \tfrac{1}{2}(u + eu)$$

$$\therefore V_P = \frac{u}{2}(1 + e) \;//$$

b) V_Q can be calculated by rearranging the equation when we used conservation of linear momentum.

$$V_Q = u - V_P = u - \frac{u}{2}(1 + e) = \frac{u}{2}(1 - e) \;//$$

c) Total energy lost $= \frac{3mu^2}{16}$

Therefore this mean the lass in kinetic energy is equal to the initial kinetic energy minus the the final kinetic energy.

$$\underbrace{\frac{3mu^2}{16}}_{\text{Lost KE}} = \underbrace{\frac{1}{2}mu^2}_{\text{Initial KE}} - \overbrace{\left[\frac{1}{2}m\left(\frac{u}{2}(1+e)\right)^2 + \frac{1}{2}m\left(\frac{u}{2}(1-e)\right)^2\right]}^{\text{Final KE}}$$

We now expand and simplify to rearrange and solve for e.

$$\therefore \frac{3mu^2}{16} = \frac{1}{2}mu^2 - \frac{1}{2}m\frac{u^2}{4}(1+e)^2 + \frac{1}{2}m\frac{u^2}{4}(1-e)^2$$

$$\frac{3mu^2}{16} = \frac{1}{4}mu^2(1-e^2)$$

$$\frac{4\times 3mu^2}{16} = mu^2(1-e^2)$$

$$\frac{12}{16} = 1-e^2 \quad \therefore \quad e^2 = 1-\frac{12}{16} = \frac{1}{4} \quad \therefore \quad e = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

However since e is only valid when $0<e<1$ $\therefore$ $e=-\frac{1}{2}$ isn't a valid solution $\therefore$ $e=\frac{1}{2}$.

d) When $e=1$, the collision is perfectly elastic therefore no kinetic energy is lost so P and Q exchange velocities and Q rebounds with the same speed. Therefore Q continues to bounce back and forth hitting P which also moves with speed u. This means the particles collide periodically, exchanging velocities, leading to repetitive motion.